COMPUTATIONAL TECHNIQUES : QUIZ-1 SOLUTIONS

PROBLEM 1

1. (a) JACOBI ITERATION

2. (b) QUADRATIC

3. (c) REGULI FALSI

4. (c) THOMAS ALGORITHM does not use partial pivoting.

5. (c) LINEAR REGRESSION

PROBLEM 2

1. $-2, -2, 3$ (−2 is REPEATED ROOT)

2. |Eigen values of Γ| < 1

3. There are two answers based on how one interprets the question.
   If 0 in sign digit is positive, else negative, we have

   <u>-1024</u> and <u>1023</u>

   If we use the full extent of sign digit, we have

   <u>-2048</u> and <u>2047</u>

   BOTH are acceptable

4. Since 11 digits are used for mantissa, PRECISION = $\underline{2.38 \times 10^{-7}}$

PROBLEM 3

1. $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & -1 & 3 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$

2. GAUSS ELIMINATION

COEFFICIENTS

STEP 1 : $R_2 - \left(\frac{1}{2}\right)R_1 \qquad R_3 - \left(\frac{1}{2}\right)R_1$

$+0.5 \leftarrow l_{21}$

$+0.5 \leftarrow l_{31}$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2.5 & 0.5 \\ 0 & -1.5 & 2.5 \end{bmatrix}$$

STEP 2 : $R_3 - \left(\frac{-1.5}{2.5}\right)R_2$

$-0.6 \leftarrow l_{32}$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2.5 & 0.5 \\ 0 & 0 & 2.8 \end{bmatrix}$$

2. & 3. THUS :

$$U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2.5 & 0.5 \\ 0 & 0 & 2.8 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.5 & -0.6 & 1 \end{bmatrix}$$

4. $L\underbrace{Ux}_{y} = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.5 & -0.6 & 1 \end{bmatrix} y = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix} \Rightarrow$$

$y_1 = 4$

$0.5y_1 + y_2 = 0 \Rightarrow y_2 = -2$

$0.5y_1 - 0.6y_2 + y_3 = 6 \Rightarrow y_3 = 2.8$

$$\therefore \underbrace{\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2.5 & 0.5 \\ 0 & 0 & 2.8 \end{bmatrix}}_{U} x = \underbrace{\begin{bmatrix} 4 \\ -2 \\ 2.8 \end{bmatrix}}_{y} \Rightarrow$$

$2.8x_3 = 2.8 \Rightarrow x_3 = 1$

$2.5x_2 + 0.5x_3 = -2 \Rightarrow x_2 = -1$

$2x_1 + x_2 + x_3 = 4 \Rightarrow x_1 = 2$

SOLUTION: $[2 \quad -1 \quad 1]^T$

PROBLEM 4

$$\frac{1}{\sqrt{f}} = -0.4 + \sqrt{3}\,\ln\left(Re\sqrt{f}\right)$$

$$x = \sqrt{f} \qquad Re = 10^{5}$$

$$F(x) \equiv \frac{1}{x} + 0.4 - \sqrt{3}\,\ln(Re) - \sqrt{3}\,\ln(x) = 0$$

$$\frac{1}{x} - 19.541 - \sqrt{3}\,\ln(x) = 0$$

$$F'(x) = -\left(\frac{1}{x^2} + \frac{\sqrt{3}}{x}\right)$$

NEWTON - RAPHSON with $f = 0.01 \Rightarrow x^{(0)} = 0.1$

ITER	x	$F(x)$	$F'(x)$	$-\frac{F(x)}{F'(x)}$
0	0.1	-5.5528	-117.32	-0.04733
1	0.05267	4.5439	-393.358	+0.01155
2	0.06422	0.7854 ~~[illegible]~~	-269.432 ~~-633.539~~	0.0029
3.	0.06714	0.0324	-247.663	1.309×10^{-4}
4.	0.06727			

$$f = x^2 = 0.01 \longrightarrow \underset{\boxed{\text{Iter 1}}}{0.00277} \longrightarrow \underset{\boxed{\text{Iter 2}}}{0.004124} \longrightarrow \underset{\boxed{\text{Iter 3}}}{0.004508} \longrightarrow \underset{\boxed{\text{Iter 4}}}{0.004525}$$

PROBLEM 5

$$r = \frac{k[S]}{K_m + [S]}$$

$$\frac{1}{r} = \frac{K_m}{k[S]} + \frac{[S]}{k[S]}$$

$$\underbrace{\frac{1}{r}}_{y} = \frac{1}{k} + \frac{K_m}{k} \underbrace{\frac{1}{[S]}}_{x}$$

$$y = a_0 + a_1 x$$

$a_0 = \frac{1}{k}$ $\quad a_1 = \frac{K_m}{k}$

$k = 1/a_0$ $\quad K_m = \frac{a_1}{a_0}$

$[S]$	0.0975	0.278	0.547	0.632	
$x = \frac{1}{[S]}$	10.256	3.5971	1.8282	1.5823	$\Rightarrow$ $\boxed{\Sigma x_i \; 17.264}$
r	0.0280	0.0531	0.0691	0.0721	
$y = \frac{1}{r}$	35.714	18.832	14.472	13.870	$\Rightarrow$ $\boxed{\Sigma y_i \; 82.888}$
x^2	105.186	12.939	3.3423	2.5037	$\Rightarrow$ $\boxed{\Sigma x_i^2 \; 123.971}$
xy	366.283	67.741	26.458	21.947	$\Rightarrow$ $\boxed{\Sigma x_i y_i \; 482.429}$

$$4a_0 + 17.264\, a_1 = 82.888$$

$$17.264\, a_0 + 123.971\, a_1 = 482.429$$

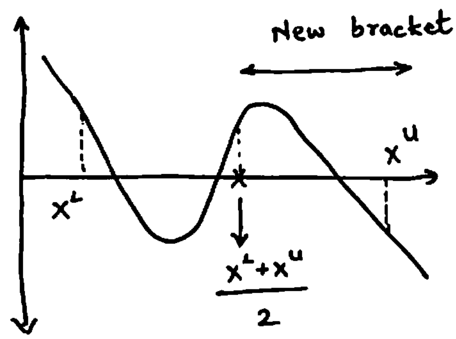
$a_0 = 9.842$ $\qquad a_1 = 2.521$

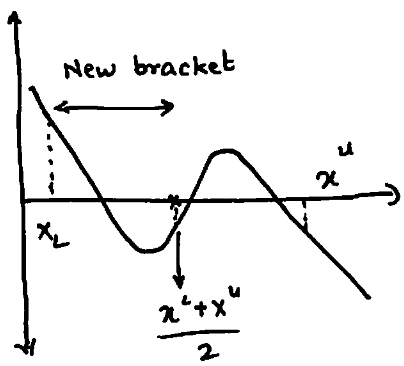
Hence $k = 0.1016$ $\qquad K_m = 0.2561$

PROBLEM 6

SAANVI IS CORRECT

For an arbitrary choice of x^L and x^u, we will be able to get $\bar{x}_1$ or $\bar{x}_3$, but usually not $\bar{x}_2$.





New bracket will either contain $\bar{x}_1$ $\bar{x}_2$ $\bar{x}_3$ or will only contain $\bar{x}_3$

New bracket will either contain $\bar{x}_1$ $\bar{x}_2$ $\bar{x}_3$ or will only contain $\bar{x}_1$

We will usually not encounter a situation where the two guesses will only bracket $\bar{x}_2$

PROBLEM 7.

$$x + 2y = 1 \Rightarrow x = 1 - 2y$$
$$x - y = 4 \Rightarrow y = x - 4$$

Gauss Siedel Method

$$x^{(k+1)} = 1 - 2y^{(k)}$$
$$y^{(k+1)} = x^{(k+1)} - 4$$

k	$x^{(k)}$	$y^{(k)}$	$1-2y^{(k)}$		$x^{(k+1)}$	$x^{(k+1)}-4$
0	1	1	-1	$\rightarrow$	-1	-5
1	-1	-5	11	$\rightarrow$	11	7
2	11	7	-21	$\rightarrow$	-21	-25
3	-21	-25	51	$\rightarrow$	51	47
4	47	51	-93	$\rightarrow$	-93	-97

GAUSS SIEDEL iterations are diverging.

In order to make them converge, the system should be made DIAGONALLY DOMINANT.

We can do so by "switching" rows 1 & 2.

PROBLEM 8

Square Root of c

$$x = c^{1/2}$$

$$f(x) \equiv x^2 - c = 0$$

$$f'(x) = 2x$$

$$x^{(i+1)} = x^{(i)} - \frac{f(x^{(i)})}{f'(x^{(i)})}$$

$$= x^{(i)} - \frac{(x^{(i)})^2}{2x^{(i)}} + \frac{c}{2x^{(i)}}$$

$$= \frac{1}{2} x^{(i)} + \frac{\cancel{K}1}{2} \frac{c}{x^{(i)}}$$

$$= \frac{1}{2}\left[x^{(i)} + \frac{c}{x^{(i)}} \right]$$

PROBLEM 9

$$c_1 = \frac{r_1}{(s_1 - s_2)(s_1 - s_3)(s_1 - s_4)} = -0.6457$$

$$c_2 = \frac{r_2}{(s_2 - s_1)(s_2 - s_3)(s_2 - s_4)} = 3.0893$$

$$c_3 = \frac{r_3}{(s_3 - s_1)(s_3 - s_2)(s_3 - s_4)} = -6.7232$$

$$c_4 = \frac{r_4}{(s_4 - s_1)(s_4 - s_2)(s_4 - s_3)} = 4.4830$$

$$P(0.5) = c_1 (0.5 - s_2)(0.5 - s_3)(0.5 - s_4) + c_2 (0.5 - s_1)(0.5 - s_3)(0.5 - s_4)$$
$$+ c_3 (0.5 - s_1)(0.5 - s_2)(0.5 - s_4) + c_4 (0.5 - s_1)(0.5 - s_2)(0.5 - s_3)$$

$$\underline{P(0.5) = 0.0673}$$

PROBLEM 10

$$g(x) = x + \beta\, G(x)$$
$$= x + \beta\left(2 - x + \ln(x)\right)$$
$$= 2\beta + x(1-\beta) + \beta \ln(x)$$

$$\frac{dg}{dx} = 1 - \beta + \frac{\beta}{x}$$

Sufficient condition for convergence $\Big\}$ $\left|1 - \beta + \frac{\beta}{x}\right| < 1$

Let us choose $\beta = 1$

$$\left|\frac{1}{x}\right| < 1 \quad \text{or} \quad |x| > 1$$

Let $x^{(0)} = 1.5$

$x^{(i)}$	$g \equiv 2 + \ln(x)$	ERROR
1.5	2.4055	
	2.8778	
	3.0570	
	3.1174	
	3.1370	
	3.1432	
	3.1452	0.0020
	3.1458	0.0006

SOLUTION : $\bar{x} = 3.1458$

PROBLEM 11

$$x^{(l)} = 2 \qquad x^{(u)} = 4$$

Iter 1 $\quad x^{(3)} = \frac{4+2}{2} = 3 \qquad \Delta x = 1$

Iter 2 $\quad x^{(4)} = 2.5$ or $3.5 \qquad \Delta x = 0.5$

Irrespective of $f(x)$

$\vdots$

Iter $n \quad \Delta x = 2^{1-n}$

We want $\Delta x \leq 10^{-3}$

$$2^{(1-n)} \leq 10^{-3}$$

$$(1-n)\log(2) \leq \log(10^{-3})$$

$$(1-n) \leq \frac{\log(10^{-3})}{\log(2)}$$

$$n \geq 1 - \frac{\log(10^{-3})}{\log(2)}$$

Hence, it will take <u>11 ITERATIONS</u> of bisection method